Name _______________ Date _______________ Period 3

Dilations

Lesson Objective IWBAT identify and perform dilations; solve real-life problems using scale factors and dilations

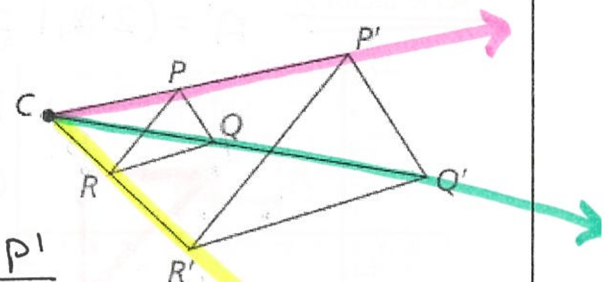
Dilation: A dilation is a transformation in which a figure is enlarged or reduced with respect to a fixed point, C, called the center of dilation and a *scale factor*, k.

→ The scale factor is the ratio of the lengths of the corresponding sides of the image and preimage

A dilation with center of dilation, C, and scale factor, k, maps every point P in a figure to P' so that the following are true:

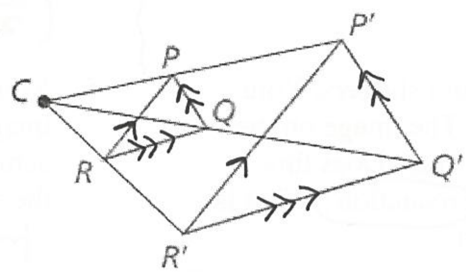
1. If P is the center point C, then P = P' same location
2. If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$
3. The scale factor, k, is a *positive* number such that: $k = \frac{\text{image}}{\text{preimage}} = \frac{CP'}{CP}$ → (also: $k = \frac{P'Q'}{PQ}$ OR $k = \frac{Q'R'}{QR}$)
4. Angle measures are preserved

$\angle P \cong \angle P'$, $\angle R \cong \angle R'$, $\angle Q \cong \angle Q'$



o in the figure above (copied again at right):

1. $\overline{PR} \parallel \overline{P'R'}$
2. $\overline{PQ} \parallel \overline{P'Q'}$
3. $\overline{QR} \parallel \overline{Q'R'}$



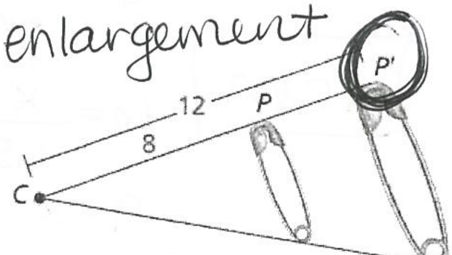
When the scale factor k > 1, the dilation is an enlargement.

When the scale factor 0 < k < 1, the dilation is an reduction.

Example:

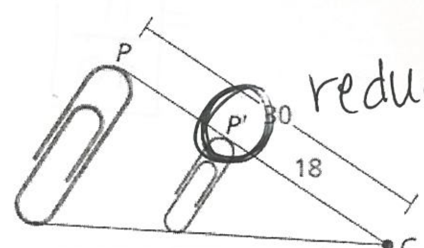
1. Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a. enlargement



$k = \frac{\text{image}}{\text{preimage}} = \frac{CP'}{CP} = \frac{12}{8} = \boxed{\frac{3}{2}}$

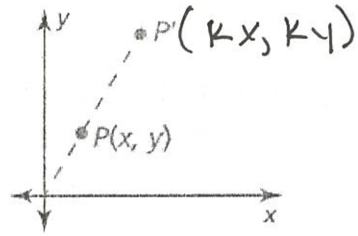
b. reduction



$k = \frac{CP'}{CP} = \frac{18}{30} = \boxed{\frac{3}{5}}$

Coordinate Rule for Dilations

If P(x, y) is the preimage of a point, then its image after a dilation centered at (0, 0) with scale factor k is the point P' (kx , ky)



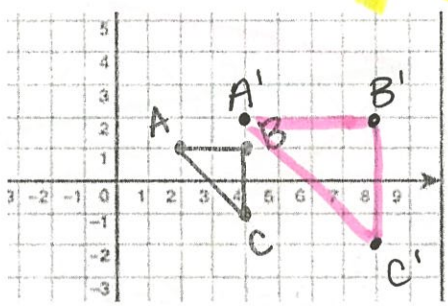
Example:

2. Graph ΔABC with vertices A(2, 1), B(4, 1) and C(4, -1) and its image after a dilation with scale factor 2.

$A' = (2 \cdot 2, 1 \cdot 2) = (4,2)$

$B'(8,2)$

$C'(8,-2)$



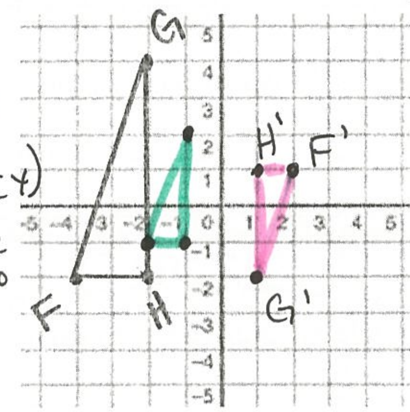
3. Graph ΔFGH with vertices F(-4, -2), G(-2, 4), and H(-2, -2) and its image after a dilation with a scale factor of $-\frac{1}{2}$.

$k = \frac{1}{2}$

$(x,y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

negative is a 180° rotation about origin

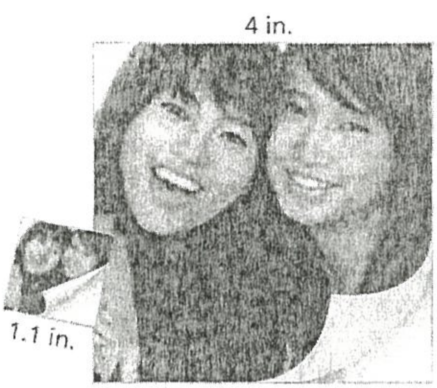
$(\frac{1}{2}x, \frac{1}{2}y) \rightarrow (-\frac{1}{2}x, -\frac{1}{2}y)$



Solving Real-Life Problems

Example:

4. You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. Does this represent an enlargement or a reduction? What is the scale factor of this dilation?

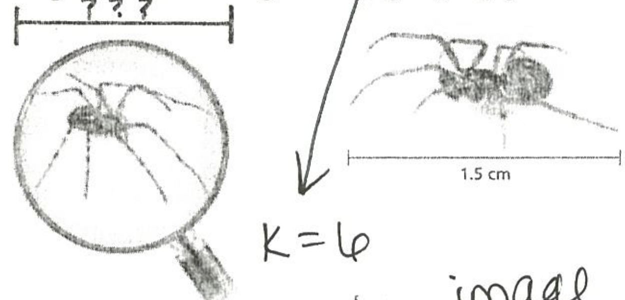


$k = \frac{\text{image}}{\text{preimage}}$

$= \frac{1.1}{4} \times \frac{10}{10}$

$= \boxed{\frac{11}{40}}$

5. You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.



$k = 6$

$6 = \frac{\text{image}}{\text{preimage}}$

$6 = \frac{x}{1.5}$

$9 = x$

$\boxed{9 \text{cm}}$

Dilations are nonrigid, since the size of the image is different than that of the preimage.

There are other nonrigid transformations, such as vertical stretches and horizontal stretches: